The Truncated Euler-Maruyama Method for Neutral Stochastic Functional Differential Equations

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Abstract

In this paper, we establish the truncated Euler-Maruyama method for neutral stochastic functional differential equations $d\left[x(t)-D(x_t)\right]=f\left(x_t,t\right)dt+g\left(x_t,t\right)dB(t)$. The key contribution is to establish the strong convergence theory of the approximate solution under the Local Lipschitz condition, Khasminskii-type condition and contractive mapping. These conditions are generally imposed to guarantee the existence and uniqueness of the true solution without the linear growth condition. The type of convergence addressed in this paper is strong-L^p convergence for $2\leq q<p$, and p is a parameter in the Khasminskii-type condition. Under some additional conditions, we will also discuss the rates of L^q-convergence of the truncated EM method for $2\leq q<p$ and show that the order of L^q-convergence can be arbitrarily close to $q/2$.

Key words: neutral stochastic functional differential equation, truncated Euler-Maruyama, strong convergence, Khasminskii-type condition.

1 Introduction

Stochastic functional differential equations (SFDEs) have been widely used in many areas. However, sometimes the systems also involve derivatives with delays. Neutral stochastic functional differential equations (NSFDEs) are used to describe such systems. For an NSFDE

$$d\left[x(t)-D\left(x_t\right)\right]=f\left(x_t,t\right)dt+g\left(x_t,t\right)dB(t),t>0$$